Texas
Educator Certification
Examination Program

Preparation Manual

Physics/Mathematics 7–12 (243)

Overview and Exam Framework

Reference Materials

Sample Selected-Response Questions

Sample Selected-Response Answers and Rationales

Section 3: Overview and Exam Framework Physics/Mathematics 7–12 (243)

Exam Overview

Exam Name	**Physics/Mathematics 7–12**
Exam Code	243
Time	5 hours
Number of Questions	120 selected-response questions
Format	Computer-administered test (CAT)

The TExES Physics/Mathematics 7–12 (243) exam is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 120 selected-response questions are based on the Physics/Mathematics 7–12 exam framework and cover grades 7–12. The exam may contain questions that do not count toward the score. Your final scaled score will be based only on scored questions.

The Standards

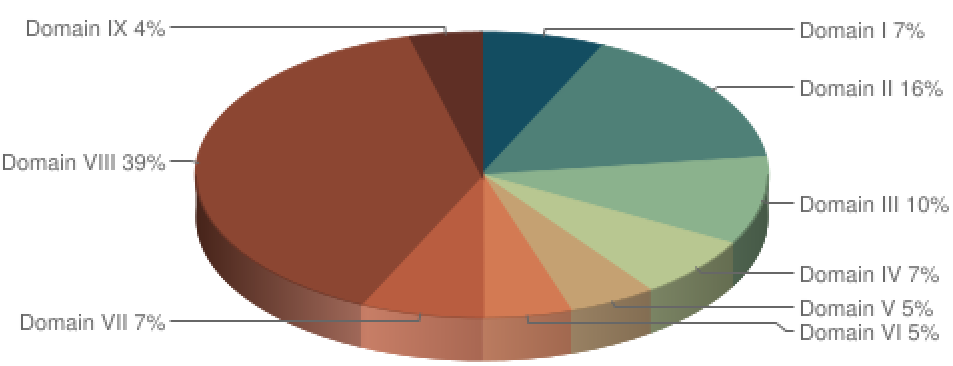
Mathematics Standard I	Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.
Mathematics Standard II	Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.
Mathematics Standard III	Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.
Mathematics Standard IV	Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Mathematics Standard V	Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.
Mathematics Standard VI	Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.
Mathematics Standard VII	Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.
Mathematics Standard VIII	Mathematical Assessment: The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.
Physical Science Standard I	The science teacher manages classroom, field and laboratory activities to ensure the safety of all students and the ethical care and treatment of organisms and specimens.
Physical Science Standard II	The science teacher understands the correct use of tools, materials, equipment and technologies.
Physical Science Standard III	The science teacher understands the process of scientific inquiry and its role in science instruction.
Physical Science Standard IV	The science teacher has theoretical and practical knowledge about teaching science and about how students learn science.
Physical Science Standard V	The science teacher knows the varied and appropriate assessments and assessment practices to monitor science learning.
Physical Science Standard VI	The science teacher understands the history and nature of science.
Physical Science Standard VII	The science teacher understands how science affects the daily lives of students and how science interacts with and influences personal and societal decisions.
Physical Science	The science teacher knows and understands the science content appropriate to teach

Standard VIII	the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in physical science.
Physical Science Standard XI	The science teacher knows unifying concepts and processes that are common to all sciences.

Domains and Competencies

Domain	Domain Title	Approx. Percentage of Exam	Standards Assessed
I	Number Concepts	7%	Mathematics I
II	Patterns and Algebra	16%	Mathematics II
III	Geometry and Measurement	10%	Mathematics III
IV	Probability and Statistics	7%	Mathematics IV
V	Mathematical Processes and Perspectives	5%	Mathematics V—VI
VI	Mathematical Learning, Instruction and Assessment	5%	Mathematics VII—VIII
VII	Scientific Inquiry and Processes	7%	Physical Science I—III, VI—VII, XI
VIII	Physics	39%	Physical Science VIII
IX	Science Learning, Instruction and Assessment	4%	Physical Science IV—V



The content covered by this exam is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of **competencies**. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

The beginning teacher:

A. Understands the concepts of place value, number base and decimal representations of real numbers and rational numbers, including benchmark fractions.

B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group, ordering of rational and real numbers).

C. Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).

D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.

E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.

F. Uses real numbers to model and solve a variety of problems.

G. Uses deductive reasoning to simplify and justify algebraic processes.

H. Demonstrates how some problems that have no solution in the integer or rational number systems have a solution in the real number system.

Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

The beginning teacher:

A. Demonstrates how some problems that have no solution in the real number system have a solution in the complex number system.

B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).

C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).

D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.

E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

The beginning teacher:

A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.

B. Applies number theory concepts and principles to justify and prove number relationships.

C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, noncommutative operations).

D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.

E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.

F. Uses estimation techniques to solve problems and judge the reasonableness of solutions.

Domain II—Patterns and Algebra

Competency 004—The teacher uses patterns to model and solve problems and formulate conjectures.

The beginning teacher:

A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.

B. Uses methods of recursion and iteration to model and solve problems.

C. Uses the principle of mathematical induction.

D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.

E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).

F. Determines the validity of logical arguments that include compound conditional statements by constructing truth tables.

Competency 005—The teacher understands attributes of functions, relations and their graphs.

The beginning teacher:

A. Understands when a relation is a function.

B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.

C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).

D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.

E. Applies basic transformations [e.g., $k\,f(x)$, $f(x) + k$, $f(x - k)$, $f(kx)$, $|f(x)|$] to a parent function, f, and describes the effects on the graph of $y = f(x)$.

F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.

G. Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x + \frac{\pi}{2})$ and $y = \cos x$].

Competency 006—The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

A. Understands the concept of slope as a rate of change, interprets the meaning of slope and intercept in a variety of situations and understands slope using similar triangles.

B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and y-intercept).

C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems and uses arrays to efficiently manage large collections of data and add, subtract and multiply matrices to solve applied problems, including geometric transformations.

D. Analyzes the zeros (real and complex) of quadratic functions.

E. Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ representations of a quadratic function and its graph.

F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).

G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.

B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.

C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or differentiable) of a function, the graph of the function and the function's symbolic representation.

D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.

E. Analyzes and applies the relationship between inverse variation and rational functions.

F. Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions, using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.

G. Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.

H. Models situations using proportional and inverse variations, including describing physical laws such as Hook's law, Newton's second law of motion and Boyle's law.

I. Uses precision and accuracy in real-life situations related to measurement and significant figures.

J. Applies and analyzes published ratings, weighted averages and indices to make informed decisions.

K. Uses proportionality to solve problems involving quantities that are not easily measured.

Competency 008—The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.

B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.

C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.

D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.

E. Models and solves problems involving exponential growth and decay.

F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.

G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).

H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = k\,f(x)$].

Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and trigonometric functions.

B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.

C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of trigonometric functions, the graphs of functions and the functions' symbolic representations.

D. Understands the relationships between trigonometric functions and their inverses and uses those relationships to solve problems.

E. Uses trigonometric identities to simplify expressions and solve equations.

F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.

G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

Competency 010—The teacher understands and solves problems using differential and integral calculus.

The beginning teacher:

A. Understands the concept of limit and the relationship between limits and continuity.

B. Relates the concepts of proportionality, rates and average rate of change and applies those concepts to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.

C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).

D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.

E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.

F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

Domain III—Geometry and Measurement

Competency 011—The teacher understands measurement as a process.

The beginning teacher:

A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.

C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.

D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

E. Relates the concept of area under a curve to the limit of a Riemann sum.

F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane and measurements associated with curves, surfaces and regions in three-space.

Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.

The beginning teacher:

A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).

B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.

C. Applies the properties of parallel and perpendicular lines to solve problems.

D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.

E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.

F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.

G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

The beginning teacher:

A. Analyzes the properties of polygons and their components.

B. Analyzes the properties of circles and the lines that intersect them.

C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).

E. Analyzes cross sections and nets of three-dimensional shapes.

F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.

G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life, including in art, architecture and music.

H. Uses similarity, geometric transformations, symmetry and perspective drawings to describe mathematical patterns and structure in architecture.

I. Uses scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and nonproportional changes in surface area and volume as applied to fields.

J. Uses the Pythagorean theorem and special right-triangle relationships to calculate distances.

K. Uses trigonometric ratios to calculate distances and angle measures as applied to fields, including using models of periodic behavior in art and music.

L. Solves geometric problems involving indirect measurement, including similar triangles, the Pythagorean theorem, law of sines, law of cosines and the use of dynamic geometry software.

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

The beginning teacher:

A. Identifies transformations (i.e., reflections, translations, glide reflections, rotations and dilations) and explores their properties.

B. Uses the properties of transformations and their compositions to solve problems.

C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.

D. Applies transformations in the coordinate plane.

E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.

F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).

G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.

H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

The beginning teacher:

A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.

B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatterplots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

C. Applies concepts of center, spread, shape and skewness to describe a data distribution.

D. Understands measures of central tendency (i.e., mean, median and mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).

E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.

F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.

G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

Competency 016—The teacher understands concepts and applications of probability.

The beginning teacher:

A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.

B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.

C. Determines probabilities by constructing sample spaces to model situations; uses a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results; calculates expected value to analyze mathematical fairness, payoff and risk.

D. Solves a variety of probability problems using combinations, permutations, and solves problems involving large quantities using combinatorics.

E. Solves a variety of probability problems using ratios of areas of geometric regions.

F. Calculates probabilities using the axioms of probability and related theorems and concepts (i.e., addition rule, multiplication rule, conditional probability, independence).

G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).

H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.

The beginning teacher:

A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.

B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.

C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).

D. Makes inferences about a population using binomial, normal and geometric distributions.

E. Describes, calculates and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).

F. Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic and power regression models.

G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.

H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).

I. Understands the principles of hypotheses testing.

J. Determines the number of ways an event may occur using combinations, permutations and the fundamental counting principle.

K. Compares theoretical to empirical probability.

L. Uses experiments to determine the reasonableness of a theoretical model (i.e., binomial, geometric).

M. Identifies limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form.

N. Interprets and compares statistical results using appropriate technology given a margin of error.

O. Identifies the variables to be used in a study.

P. Analyzes possible sources of data variability, including those that can be controlled and those that cannot be controlled.

Q. Reports results of statistical studies to a particular audience by selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

The beginning teacher:

A. Understands the nature of proof, including indirect proof, in mathematics.

B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.

C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.

D. Uses formal and informal reasoning to justify mathematical ideas.

E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).

F. Evaluates how well a mathematical model represents a real-world situation.

Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

The beginning teacher:

A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).

B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).

C. Translates mathematical ideas between verbal and symbolic forms.

D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

E. Understands the use of visual media (e.g., graphs, tables, diagrams, animations) to communicate mathematical information.

F. Uses appropriate mathematical terminology to express mathematical ideas.

G. Explores and applies concepts of financial literacy as it relates to teaching students (e.g., describes the basic purpose of financial institutions, distinguishes the difference between gross income and net income, identifies various savings options, defines different types of taxes, identifies the advantages and disadvantages of different methods of payment).

H. Applies mathematics to model and solve problems to manage financial resources effectively for lifetime financial security (e.g., distinguishes between fixed and variable expenses, calculates profit in a given situation, develops a system for keeping and using financial records, describes actions that might be taken to balance a budget when expenses exceed income, balances a simple budget).

I. Analyzes various voting and selection processes to compare results in given situations, selects and applies an algorithm of interest to solve real-life problems (e.g., using recursion or iteration to calculate population growth or decline, fractals or compound interest; determining validity in recorded and transmitted data using checksums and hashing; evaluating sports rankings, weighted class rankings and search-engine rankings; solving problems involving scheduling or routing using vertex-edge graphs, critical paths, Euler paths or minimal spanning trees), and communicates to peers the application of the algorithm in precise mathematical and nontechnical language.

J. Determines or analyzes an appropriate cyclical model for problem situations that can be modeled with periodic functions; determines or analyzes an appropriate piecewise model for problem situations; creates, represents and analyzes mathematical models for various types of income calculations to determine the best option for a given situation; creates, represents and analyzes mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; creates, represents and analyzes mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 020—The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

The beginning teacher:

A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.

B. Understands how students differ in their approaches to learning mathematics.

C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.

D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stopwatches, scales, rulers).

E. Understands how to provide instruction along a continuum from concrete to abstract.

F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.

G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.

H. Understands a variety of questioning strategies to encourage mathematical discourse and help students analyze and evaluate their mathematical thinking.

I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

The beginning teacher:

A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.

B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.

C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.

D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

The beginning teacher:

A. Uses current sources of information about laboratory safety, including safety regulations and guidelines for the use of science facilities.

B. Recognizes potential safety hazards in the laboratory and in the field and knows how to apply procedures, including basic first aid, for responding to accidents.

C. Employs safe practices in planning, implementing and managing all instructional activities and designs and implements rules and procedures to maintain a safe learning environment.

D. Understands procedures for selecting, maintaining and safely using chemicals, tools, technologies, materials, specimens and equipment, including procedures for the recycling, reuse and conservation of laboratory resources and for the safe handling and ethical treatment of organisms.

E. Knows how to use appropriate equipment and technology (e.g., Internet, spreadsheet, calculator) for gathering, organizing, displaying and communicating data in a variety of ways (e.g., charts, tables, graphs, diagrams, maps, satellite images, written reports, oral presentations).

F. Understands how to use a variety of tools, techniques and technology to gather, organize and analyze data and perform calculations and knows how to apply appropriate methods of statistical measures and analysis.

G. Knows how to apply techniques to calibrate measuring devices and understands concepts of precision, accuracy and error with regard to reading and recording numerical data from scientific instruments (e.g., significant figures).

H. Uses the International System of Units (i.e., metric system) and performs unit conversions within and across measurement systems.

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

The beginning teacher:

A. Understands the nature of science, the relationship between science and technology, the predictive power of science and limitations to the scope of science (i.e., the types of questions that science can and cannot answer).

B. Knows the characteristics of various types of scientific investigations (e.g., descriptive studies, controlled experiments, comparative data analysis) and how and why scientists use different types of scientific investigations.

C. Understands principles and procedures for designing and conducting a variety of scientific investigations, with emphasis on inquiry-based investigations, and knows how to communicate and defend scientific results.

D. Understands how logical reasoning, verifiable observational and experimental evidence and peer review are used in the process of generating and evaluating scientific knowledge.

E. Understands how to identify potential sources of error in an investigation, evaluate the validity of scientific data and develop and analyze different explanations for a given scientific result.

F. Knows the characteristics and general features of systems, how properties and patterns of systems can be described in terms of space, time, energy and matter and how system components and different systems interact.

G. Knows how to apply and analyze the systems model (e.g., interacting parts, boundaries, input, output, feedback, subsystems) across the science disciplines.

H. Understands how shared themes and concepts (e.g., systems, order and organization; evidence, models and explanation; change, constancy and measurements; evolution and equilibrium; and form and function) provide a unifying framework in science.

I. Understands the difference between a theory and a hypothesis, how models are used to represent the natural world and how to evaluate the strengths and limitations of a variety of scientific models (e.g., physical, conceptual, mathematical).

Competency 024—The teacher understands the history of science, how science impacts the daily lives of students and how science interacts with and influences personal and societal decisions.

The beginning teacher:

A. Understands the historical development of science, key events in the history of science and the contributions that diverse cultures and individuals of both genders have made to scientific knowledge.

B. Knows how to use examples from the history of science to demonstrate the changing nature of scientific theories and knowledge (i.e., that scientific theories and knowledge are always subject to revision in light of new evidence).

C. Knows that science is a human endeavor influenced by societal, cultural and personal views of the world and that decisions about the use and direction of science are based on factors such as ethical standards, economics and personal and societal biases and needs.

D. Understands the application of scientific ethics to the conducting, analyzing and publishing of scientific investigations.

E. Applies scientific principles to analyze factors (e.g., diet, exercise, personal behavior) that influence personal and societal choices concerning fitness and health (e.g., physiological and psychological effects and risks associated with the use of substances and substance abuse).

F. Applies scientific principles, the theory of probability and risk/benefit analysis to analyze the advantages of, disadvantages of or alternatives to a given decision or course of action.

G. Understands the role science can play in helping resolve personal, societal and global issues (e.g., recycling, population growth, disease prevention, resource use, evaluating product claims).

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

The beginning teacher:

A. Generates, analyzes and interprets graphs describing the motion of a particle.

B. Applies vector concepts to displacement, velocity and acceleration in order to analyze and describe the motion of a particle.

C. Solves problems involving uniform and accelerated motion using scalar (e.g., speed) and vector (e.g., velocity) quantities.

D. Analyzes and solves problems involving projectile motion.

E. Analyzes and solves problems involving uniform circular and rotary motion.

F. Understands motion of fluids.

G. Understands motion in terms of frames of reference and relativity concepts.

Competency 026—The teacher understands the laws of motion.

The beginning teacher:

A. Identifies and analyzes the forces acting in a given situation and constructs a free-body diagram.

B. Solves problems involving the vector nature of force (e.g., resolving forces into components, analyzing static or dynamic equilibrium of a particle).

C. Identifies and applies Newton's laws to analyze and solve a variety of practical problems (e.g., properties of frictional forces, acceleration of a particle on an inclined plane, displacement of a mass on a spring, forces on a pendulum).

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

The beginning teacher:

A. Applies the law of universal gravitation to solve a variety of problems (e.g., determining the gravitational fields of the planets, analyzing properties of satellite orbits).

B. Calculates electrostatic forces, fields and potentials.

C. Understands the properties of magnetic materials and the molecular theory of magnetism.

D. Identifies the source of the magnetic field and calculates the magnetic field for various simple current distributions.

E. Analyzes the magnetic force on charged particles and current-carrying conductors.

F. Understands induced electric and magnetic fields and analyzes the relationship between electricity and magnetism.

G. Understands the electromagnetic spectrum and the production of electromagnetic waves.

Competency 028—The teacher understands applications of electricity and magnetism.

The beginning teacher:

A. Analyzes common examples of electrostatics (e.g., a charged balloon attached to a wall, behavior of an electroscope, charging by induction).

B. Understands electric current, resistance and resistivity, potential difference, capacitance and electromotive force in conductors and circuits.

C. Analyzes series and parallel DC circuits in terms of current, resistance, voltage and power.

D. Identifies basic components and characteristics of AC circuits.

E. Understands the operation of an electromagnet.

F. Understands the operation of electric meters, motors, generators and transformers.

Competency 029—The teacher understands the conservation of energy and momentum.

The beginning teacher:

A. Understands the concept of work.

B. Understands the relationships among work, energy and power.

C. Solves problems using the conservation of mechanical energy in a physical system (e.g., determining potential energy for conservative forces, conversion of potential to kinetic energy, analyzing the motion of a pendulum).

D. Applies the work-energy theorem to analyze and solve a variety of practical problems (e.g., finding the speed of an object given its potential energy, determining the work done by frictional forces on a decelerating car).

E. Understands linear and angular momentum.

F. Solves a variety of problems (e.g., collisions) using the conservation of linear and angular momentum.

Competency 030—The teacher understands the laws of thermodynamics.

The beginning teacher:

A. Understands methods of heat transfer (i.e., convection, conduction, radiation).

B. Understands the molecular interpretation of temperature and heat.

C. Solves problems involving thermal expansion, heat capacity and the relationship between heat and other forms of energy.

D. Applies the first law of thermodynamics to analyze energy transformations in a variety of everyday situations (e.g., electric light bulb, power-generating plant).

E. Understands the concept of entropy and its relationship to the second law of thermodynamics.

Competency 031—The teacher understands the characteristics and behavior of waves.

The beginning teacher:

A. Understands interrelationships among wave characteristics such as velocity, frequency, wavelength and amplitude and relates them to properties of sound and light (e.g., pitch, color).

B. Compares and contrasts transverse and longitudinal waves.

C. Describes how various waves are propagated through different media.

D. Applies properties of reflection and refraction to analyze optical phenomena (e.g., mirrors, lenses, fiber-optic cable).

E. Applies principles of wave interference to analyze wave phenomena, including acoustical (e.g., harmonics) and optical phenomena (e.g., patterns created by thin films and diffraction gratings).

F. Identifies and interprets how wave characteristics and behaviors are used in medical, industrial and other real-world applications.

Competency 032—The teacher understands the fundamental concepts of quantum physics.

The beginning teacher:

A. Interprets wave-particle duality.

B. Identifies examples and consequences of the uncertainty principle.

C. Understands the photoelectric effect.

D. Understands the quantum model of the atom and can use it to describe and analyze absorption and emission spectra (e.g., line spectra, blackbody radiation) and other phenomenon (e.g., radioactive decay, nuclear forces, nuclear reactions).

E. Explores real-world applications of quantum phenomena (e.g., lasers, photoelectric sensors, semiconductors, superconductivity).

Domain IX—Science Learning, Instruction and Assessment

Competency 033—The teacher understands researched-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

The beginning teacher:

A. Knows research-based theories about how students develop scientific understanding and how developmental characteristics, prior knowledge, experience and attitudes of students influence science learning.

B. Understands the importance of respecting student diversity by planning activities that are inclusive and selecting and adapting science curricula, content, instructional materials and activities to meet the interests, knowledge, understanding, abilities, possible career paths and experiences of all students, including English-language learners.

C. Knows how to plan and implement strategies to encourage student self-motivation and engagement in their own learning (e.g., linking inquiry-based investigations to students' prior knowledge, focusing inquiry-based instruction on issues relevant to students, developing instructional materials using situations from students' daily lives, fostering collaboration among students).

D. Knows how to use a variety of instructional strategies to ensure all students comprehend content-related texts, including how to locate, retrieve and retain information from a range of texts and technologies.

E. Understands the science teacher's role in developing the total school program by planning and implementing science instruction that incorporates schoolwide objectives and the statewide curriculum as defined in the Texas Essential Knowledge and Skills (TEKS).

F. Knows how to design and manage the learning environment (e.g., individual, small-group, whole-class settings) to focus and support student inquiries and to provide the time, space and resources for all students to participate in field, laboratory, experimental and nonexperimental scientific investigation.

G. Understands the rationale for using active learning and inquiry methods in science instruction and how to model scientific attitudes such as curiosity, openness to new ideas and skepticism.

H. Knows principles and procedures for designing and conducting an inquiry-based scientific investigation (e.g., making observations; generating questions; researching and reviewing current knowledge in light of existing evidence; choosing tools to gather and analyze evidence; proposing answers, explanations and predictions; and communicating and defending results).

I. Knows how to assist students with generating, refining, focusing and testing scientific questions and hypotheses.

J. Knows strategies for assisting students in learning to identify, refine and focus scientific ideas and questions guiding an inquiry-based scientific investigation; to develop, analyze and evaluate different explanations for a given scientific result; and to identify potential sources of error in an inquiry-based scientific investigation.

K. Understands how to implement inquiry strategies designed to promote the use of higher-level thinking skills, logical reasoning and scientific problem solving in order to move students from concrete to more abstract understanding.

L. Knows how to guide students in making systematic observations and measurements.

M. Knows how to sequence learning activities in a way that uncovers common misconceptions, allows students to build upon their prior knowledge and challenges them to expand their understanding of science.

Competency 034—The teacher knows how to monitor and assess science learning in laboratory, field and classroom settings.

The beginning teacher:

A. Knows how to use formal and informal assessments of student performance and products (e.g., projects, laboratory and field journals, rubrics, portfolios, student profiles, checklists) to evaluate student participation in and understanding of inquiry-based scientific investigations.

B. Understands the relationship between assessment and instruction in the science curriculum (e.g., designing assessments to match learning objectives, using assessment results to inform instructional practice).

C. Knows the importance of monitoring and assessing students' understanding of science concepts and skills on an ongoing basis by using a variety of appropriate assessment methods (e.g., performance assessment, self-assessment, peer assessment, formal/informal assessment).

D. Understands the purposes, characteristics and uses of various types of assessment in science, including formative and summative assessments, and the importance of limiting the use of an assessment to its intended purpose.

E. Understands strategies for assessing students' prior knowledge and misconceptions about science and how to use those assessments to develop effective ways to address the misconceptions.

F. Understands characteristics of assessments, such as reliability, validity and the absence of bias in order to evaluate assessment instruments and their results.

G. Understands the role of assessment as a learning experience for students and strategies for engaging students in meaningful self-assessment.

H. Recognizes the importance of selecting assessment instruments and methods that provide all students with adequate opportunities to demonstrate their achievements.

I. Recognizes the importance of clarifying teacher expectations by sharing evaluation criteria and assessment results with students.

This reference material will also be available to you during the exam. To access it, click on the

Reference Materials icon located in the lower-left corner of the screen.

Definitions and Formulas

CALCULUS

First Derivative: $f'(x) = \dfrac{dy}{dx}$

Second Derivative: $f''(x) = \dfrac{d^2y}{dx^2}$

PROBABILITY

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$

ALGEBRA

i — $i^2 = -1$

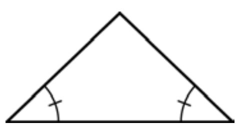
$\boldsymbol{A}^{-1}$ — inverse of matrix A

$\boldsymbol{A} = \boldsymbol{P}\left(\mathbf{1} + \dfrac{\boldsymbol{r}}{\boldsymbol{n}}\right)^{\boldsymbol{nt}}$ — Compound interest,
where A is the final value
P is the principal
r is the interest rate
t is the term
n is the number of divisions within the term

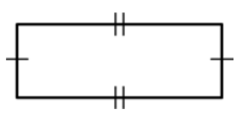
$[\boldsymbol{x}] = \boldsymbol{n}$ — Greatest integer function,
where n is the integer such that $n \le x < n + 1$

GEOMETRY

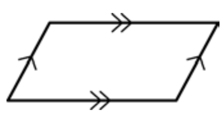
Congruent Angles



Congruent Sides



Parallel Sides



Circumference of a Circle

$C = 2\pi r$

VOLUME

Cylinder: (area of base) $\times$ height

Cone: $\dfrac{1}{3}$(area of base) $\times$ height

Sphere: $\dfrac{4}{3}\pi r^3$

Prism: (area of base) $\times$ height

AREA

Triangle: $\dfrac{1}{2}(\text{base} \times \text{height})$

Rhombus: $\dfrac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

Trapezoid: $\dfrac{1}{2}\text{height }(\text{base}_1 + \text{base}_2)$

Sphere: $4\pi r^2$

Circle: πr^2

Lateral surface area of cylinder: $2\pi rh$

TRIGONOMETRY

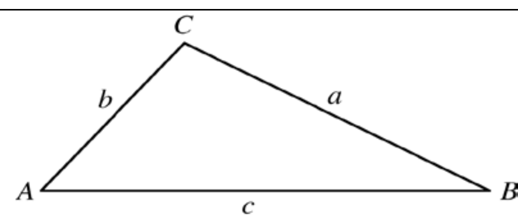
Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$



End of Definitions and Formulas

This reference material will also be available to you during the exam. To access it, click on the Reference Materials icon located in the lower-left corner of the screen.

Definitions and Physical Constants

The value of 9.8 m/s^2 is used for the acceleration of gravity near Earth's surface.

The universal gas constant is 8.314 J/K-mol or 0.08206 L-atm/K-mol.

Planck's constant is 6.6256×10^{-34} J-s.

Avogadro's number is 6.022×10^{23}.

The right-hand rule is used with conventional current (the flow of positive charge from the positive terminal to the negative terminal).

End of Definitions and Physical Constants

Section 4: Sample Selected-Response Questions Physics/Mathematics 7–12 (243)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 23)
- Definitions and Physical Constants (see page 24)

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

1. A light year is the distance light travels in one year. The star Procyon is approximately 1.08×10^{17} meters from our Sun. If light travels at 3.00×10^{8} meters per second, and if there are approximately 3.16×10^{7} seconds in a year, what is the approximate distance, in light years, from Procyon to our Sun?

   A. 11.39

   B. 87.78

   C. 1.14×10^{2}

   D. 8.78×10^{-2}

Answer _____

2. The number of bacteria in a colony increases from 400 to 800 during 1 hour. What is the rate of growth?

   A. 400% per hour

   B. 200% per hour

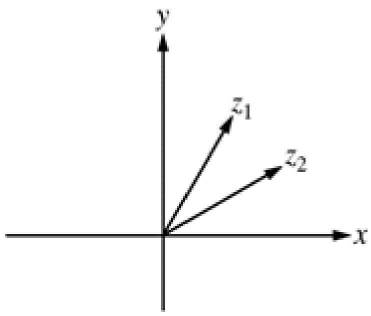
   C. 100% per hour

   D. 50% per hour

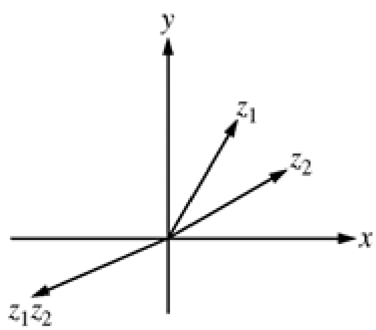
Answer _____

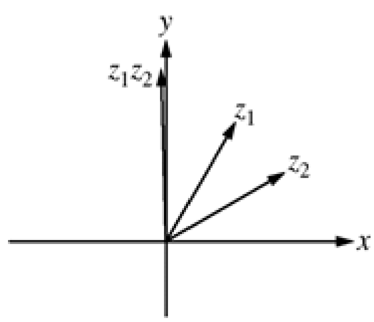
Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

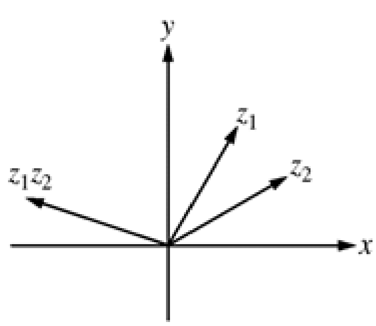
Use the figure below to answer the question that follows.

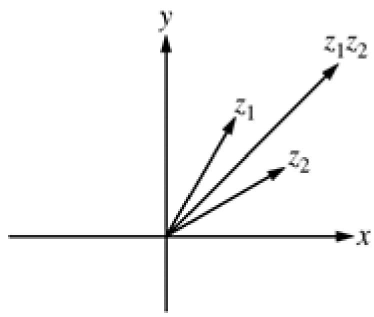


3. Two complex numbers, Z_1 and Z_2 are shown in the complex plane above. Which of the following graphs could depict the product Z_1Z_2 in the complex plane?

A. 

B. 

C. 



D.

Answer _____

Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

4. Given that $n - 5$ and $n + 6$ are positive integers both divisible by prime number p, which of the following integers must also be divisible by p?

A. $n + 1$

B. $n - 30$

C. $n + 28$

D. $n - 11$

Answer _____

5. On the first day of 2011 Asad deposited $500 into a savings account earning 3 percent annual interest, compounded annually at the end of the year. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2012, Asad deposited an additional $500 into the account, earning the same annual interest, compounded annually. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2013, Asad deposited an additional $500 into the account, earning the same annual interest, compounded annually. If no additional deposits or withdrawals were made, which of the following expressions represents the amount of money in Asad's account at the end of 2013?

A. $3[500 + 500(0.03)] + 500(0.06)$

B. $3[500 + 500(1.03) + 500(2.06)]$

C. $500(1.03) + 500(1.03)^2 + 500(1.03)^3$

D. $500(0.03) + 500(0.03)^2 + 500(0.03)^3$

Answer _____

Domain II—Patterns and Algebra

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the equations below to answer the question that follows.

$$y = -3x - 2$$
$$y = \begin{cases} x + 2, & x \le 2 \\ 10 - 2x, & x > 2 \end{cases}$$

6. Which of the following are the points of intersection of the graphs in the *xy*-plane of the equations shown?

A. (–1, 1) only

B. (–1, –5) only

C. (–12, 34) only

D. (–1, 1) and (–12, 34)

Answer _____

Competency 008—The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

7. Which of the following is a vertical asymptote of the graph of $y = 3 + \log_2(x - 4)$?

A. $y = 3$

B. $y = -4$

C. $x = -4$

D. $x = 4$

Answer _____

Competency 010—The teacher understands and solves problems using differential and integral calculus.

8. $\frac{d}{dx}\int_0^x \sin t \cos t \, dt =$

A. $\frac{\sin^2 x \cos^2 x}{4}$

B. $\cos 2x$

C. $\frac{\sin^2 x}{2}$

D. $\sin x \cos x$

Answer _____

Domain III—Geometry and Measurement

Competency 011—The teacher understands measurement as a process.

9. Which of the following definite integrals represents the length of the curve $y = x^2 + 1$ between the points (0, 1) and (2, 5)?

A. $\int_0^2 (1 + 4x^2)\,dx$

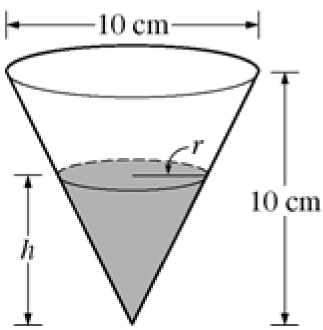
B. $\int_0^2 \sqrt{1 + 4x^2}\,dx$

C. $\int_0^2 \sqrt{1 + x^2}\,dx$

D. $\pi\int_0^2 4x^2\,dx$

Answer _____

Use the figure below to answer the question that follows.



10. An inverted circular cone contains water, as shown. If the height, *h*, of the water is 8 centimeters, what is the radius, *r*, of the surface of the water, in centimeters?

A. 8

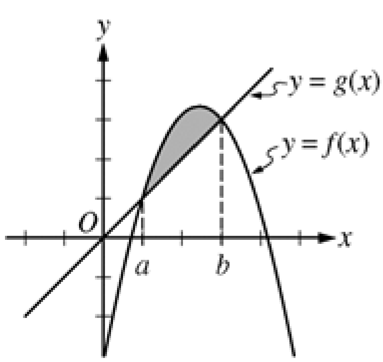
B. 6

C. 4

D. 2

Answer _____

Use the figure below to answer the question that follows.



11. Let *f* and *g* be continuous functions defined on the interval [*a*, *b*], as shown. Which of the following can be used to find the area of the shaded region?

A. $\int_a^b f(x)dx - \int_a^b g(x)dx$

B. $\int_a^b g(x)dx - \int_b^a f(x)dx$

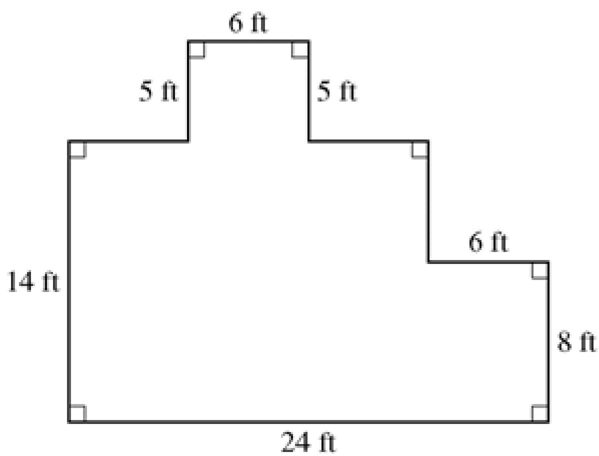
C. $\int_a^b \pi (f(x) - g(x))^2 \, dx$

D. $2\pi \int_a^b x(f(x) - g(x))\, dx$

Answer _____

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

Use the figure below to answer the question that follows.



12. The floor plan of Nick's den is shown. Nick wishes to purchase square tiles that are 12 inches on each side to tile the entire floor. To allow for possible breakage, he plans to purchase more tiles to cover an additional 10 percent of the floor area. How many tiles will Nick purchase?

A. 330 tiles

B. 363 tiles

C. 366 tiles

D. 402 tiles

Answer _____

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

13. Let the vectors *i* and *j* be given by *i* = (1, 0) and *j* = (0, 1). A person walks 80 feet due east, and then turns counter-clockwise 40° and walks an additional 60 feet in that direction. Which of the following vectors describes the person's final position relative to his or her starting position?

A. $(60\cos40°)i + (80 + 60\cos40°)j$

B. $(60\cos40°)i + (80 + 60\sin40°)j$

C. $(80 + 60\sin40°)i + (60\sin40°)j$

D. $(80 + 60\cos40°)i + (60\sin40°)j$

Answer _____

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

Use the chart below to answer the question that follows.

SALARIES OF STUDENTS WORKING PART-TIME

14. The bar graph shown represents the distributions of weekly salaries, rounded to the nearest dollar, for a group of students with part-time jobs. Which of the following statements is true?

A. The range of the weekly salaries is $499.

B. The median weekly salary is between $240 and $260.

C. The average weekly salary is between $167 and $267.

D. The mode of the weekly salaries is between $200 and $299.

Answer _____

Competency 016—The teacher understands concepts and applications of probability.

15. Mr. Allen's math class consists of 15 boys and 15 girls. Each day he randomly selects 3 students to present homework problems on the blackboard. What is the probability that the three chosen students will be all boys? (Note: ${}_nC_r = \frac{n!}{(n-r)!r!}$ denotes combinations of n objects taken r at a time, and ${}_nP_r = \frac{n!}{(n-r)!}$ denotes permutations of n objects taken r at a time.)

A. $\frac{{}_{15}P_3}{{}_{30}C_3}$

B. $\dfrac{_{15}C_3}{_{15}P_3}$

C. $\dfrac{_{15}C_3}{_{30}C_3}$

D. $\dfrac{_{15}P_3}{_{15}C_3}$

Answer _____

Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.

16. A packing company packages approximately 10,000 packages of noodles each day. The packages have a mean weight of 16 ounces with a standard deviation of 0.2 ounces. An inspector selects multiple samples of 25 packages each and computes the mean weight of each sample. Which of the following is the best estimate of the standard deviation of the mean weights of the samples?

A. 0.02

B. 0.04

C. 0.2

D. 1.0

Answer _____

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

17. Mr. Matthews' precalculus class is studying even and odd functions. Mr. Matthews gave a function to a group of students in his class and asked them to investigate its symmetry without the use of a graphing calculator. Katie, Shayan and Diego are given the function $f(x) = \dfrac{x^3 + x}{|x| - 4}$, where $x \neq \pm 4$. Katie notes that $f(1) = -\dfrac{2}{3}$ and $f(-1) = \dfrac{2}{3}$. Shayan computes $f(3) = -30$ and $f(-3) = 30$. Diego discovers that $f(2) = -5$ and $f(-2) = 5$. After comparing notes, the three students conjecture that the function is odd because the graph exhibits symmetry with respect to the origin. Which of the following activities would enable the group to verify this conjecture?

A. The students could look for a counterexample by trying several values for *x*. If they are not able to find a counterexample, they can deduce that the function exhibits symmetry with respect to the origin and is therefore odd.

B. The students could evaluate the function at an additional eight pairs of values of *x* that are additive inverses and note that in each case, $f(-x) = -f(x)$. They could then deduce that the conjecture is true.

C. The students could simplify the algebraic expression for $f(-x)$ and establish that it is equal to $-f(x)$ for all values of x, $x \neq \pm 4$.

D. The students could simplify the algebraic expression for $f(-x)$ and establish that it is equal to $f(x)$ for all values of x, $x \neq \pm 4$.

Answer _____

Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

18. Let C be the set of all right-circular cylinders of radius 3. Let $v(x)$ be the volume of the cylinders in C as a function of height, $x \geq 0$. The graph of the function $y = v(x)$ in the xy-plane is

A. a horizontal line through the point (0, 9π).

B. a line with slope 9π and y-intercept 0.

C. a parabola with vertex (0, 0) passing through the point (1, 9π).

D. a cubic polynomial passing through the points (0, 0), (1, 9π), and (3, 27π).

Answer _____

19. Bank A and Bank B each offer 5 percent interest rate on savings accounts, but Bank A compounds the interest annually and Bank B compounds the interest semiannually. If Samantha invests $10,000 in a savings account in Bank B, and makes no other transactions in her account, how much more interest will she earn in a year than she would have earned if she had invested the money in Bank A?

A. $ 5.00

B. $ 6.25

C. $ 7.25

D. $ 8.50

Answer _____

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

20. Which of the following is a main purpose of formative assessments?

A. To provide feedback for teachers and students to improve the teaching and learning activities used for instruction

B. To summarize and record overall achievement at the end of an instructional unit

C. To evaluate the effectiveness of the curriculum

D. To identify students ready for promotion to the next level of a course

Answer _____

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

21. The standard deviation of a set of data measurements is related to which of the following? Select all that apply.

A. Precision

B. Measurement repeatability

C. Systematic error

D. Reproducibility

Answer _____

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

22. Scientific theories are based on which of the following? Select all that apply.

A. Observations

B. Experiments

C. Empirical generalizations

D. Personal beliefs and speculations

Answer _____

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

23. A ball of mass *m* is thrown horizontally with an initial speed u_0 from the top of a building that is height *h* above level ground. In the absence of air resistance, what is the speed of the ball when it strikes the ground?

A. u_0

B. $\sqrt{2gh}$

C. $u_0 + \sqrt{2gh}$

D. $\sqrt{u_0^2 + 2gh}$

Answer _____

24. Two satellites are each in a circular orbit around Earth at distances *R* and 2*R*, respectively, from Earth's center. If the satellite at distance *R* has an orbital period equal to *T*, the satellite at distance 2*R* must have an orbital period equal to

A. $2\sqrt{2}\,T$

B. $2T$

C. T

D. $\frac{1}{2}T$

Answer _____

25. A block of mass 2 kg and volume 4×10^{-4} m^3 is suspended from a thin wire and completely submerged in water. What is the tension in the wire? (Assume g = 10 m/s^2.)

A. 0

B. 4 N

C. 16 N

D. 20 N

Answer _____

Competency 026—The teacher understands the laws of motion.

26. A 40 kg box is pulled up a long, frictionless ramp by a 400 N force. If the ramp is inclined at an angle of 30° with respect to the horizontal, what is the magnitude of the acceleration of the block? (Assume g = 10 m/s^2.)

A. 0

B. 5 m/s^2

C. 10 m/s^2

D. 15 m/s^2

Answer _____

27. A 50 N force is being applied to a 10 kg block on a rough horizontal surface. If the coefficient of kinetic friction between the moving block and the surface is 0.25, what is the magnitude of the acceleration of the block? (Assume g = 10 m/s^2.)

A. 2.5 m/s^2

B. 5 m/s^2

C. 7.5 m/s^2

D. 10 m/s^2

Answer _____

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

28. A simple pendulum of length L has a period T on Earth. If a simple pendulum is to have the same period T on Jupiter, where the gravitational force is 2.5 times as great as the gravitational force on Earth, what does the length of the pendulum have to be?

A. $2.5L$

B. $\sqrt{2.5}\,L$

C. L

D. $\frac{L}{2.5}$

Answer _____

29. A planet moves in an elliptical orbit about a star. Which of the following is true about the orbital speed of the planet?

A. It is constant.

B. It varies and is greatest when the planet is closest to the star.

C. It varies and is greatest when the planet is farthest from the star.

D. It cannot be determined from known physical laws.

Answer _____

30. A coil of wire produces a magnetic field of magnitude B. If the number of turns per unit length in the coil is increased by a factor of 2, the magnitude of the magnetic field produced by the coil will be equal to which of the following?

A. $\frac{1}{2}B$

B. B

C. $2B$

D. $4B$

Answer _____

Competency 028—The teacher understands applications of electricity and magnetism.

31. A 4 Ω resistor and a 12 Ω resistor are connected in parallel to a 12 V battery. The power dissipated by the circuit is equal to which of the following?

A. 48 W

B. 18 W

C. 9 W

D. 4 W

Answer _____

32. Which of the following uses mechanical energy to produce electrical energy?

A. A transformer

B. A motor

C. A generator

D. A battery

Answer _____

Competency 029—The teacher understands the conservation of energy and momentum.

33. A railroad boxcar of mass *M* is moving at speed *u* along a straight horizontal track. It collides elastically with a second boxcar of the same mass *M* that is at rest. What are the magnitude of the total momentum and the total kinetic energy of the two boxcars immediately after the collision?

	Momentum	Kinetic Energy
A.	0	$\frac{1}{2}Mu^2$
B.	Mu	$\frac{1}{2}Mu^2$
C.	0	Mu^2
D.	Mu	Mu^2

Answer _____

Competency 030—The teacher understands the laws of thermodynamics.

34. The second law of thermodynamics is specifically concerned with which of the following?

A. The conservation of energy

B. The behavior of gases

C. The behavior of the entropy of a system

D. The molecular basis of temperature

Answer _____

35. When 100 J of heat energy is added to a system, the system does 40 J of work. What is the increase in the internal energy of the system?

A. 40 J

B. 60 J

C. 100 J

D. 140 J

Answer _____

Competency 031—The teacher understands the characteristics and behavior of waves.

36. A light ray passes from air (n = 1) into glass (n = 1.5). If the incident angle is 30°, what is the angle of refraction?

A. 30°

B. 60°

C. $\sin^{-1}\frac{1}{3}$

D. $\sin^{-1}\frac{3}{4}$

Answer _____

37. Which of the following is characteristic of light but not of sound?

A. Dispersion

B. Constructive and destructive interference

C. Bending around an obstacle

D. Propagation in a vacuum

Answer _____

Competency 032—The teacher understands the fundamental concepts of quantum physics.

38. According to the Bohr model of the hydrogen atom, how does the energy of an electron depend on the principal quantum number n?

A. The energy of an electron is proportional to $\frac{1}{n^2}$.

B. The energy of an electron is proportional to $\frac{1}{n}$.

C. The energy of an electron is proportional to n.

D. The energy of an electron is proportional to n^2.

Answer _____

39. In the photoelectric effect, light is incident on the surface of a metal and causes photoelectrons to be emitted. For a given metal, which of the following is true about the energy of the emitted photoelectrons?

A. The energy of the photoelectrons increases when the frequency of the incident light increases.

B. The energy of the photoelectrons increases when the intensity of the incident light increases.

C. The energy of the photoelectrons increases when the wavelength of the incident light increases.

D. The energy of the photoelectrons is characteristic of the metal and independent of the incident light.

Answer _____

Domain IX—Science Learning, Instruction and Assessment

Competency 033—The teacher understands researched-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

40. Which of the following student responses is an example of correct conceptual understanding?

   A. The mass of an object is equal to its weight.

   B. The speed of light is greater than the speed of sound.

   C. Heavy objects always sink in water.

   D. The Moon and the Sun are the same size.

Answer _____

Section 4: Sample Selected-Response Answers and Rationales
Physics/Mathematics 7–12 (243)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 23)
- Definitions and Physical Constants (see page 24)

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

1. A light year is the distance light travels in one year. The star Procyon is approximately 1.08×10^{17} meters from our Sun. If light travels at 3.00×10^{8} meters per second, and if there are approximately 3.16×10^{7} seconds in a year, what is the approximate distance, in light years, from Procyon to our Sun?

A. 11.39

B. 87.78

C. 1.14×10^{2}

D. 8.78×10^{-2}

Answer

Option A is correct because the distance light travels in one year is $(3.00 \times 10^{8}$ m/sec$)(3.16 \times 10^{7}$ sec/yr$) = 9.48 \times 10^{15}$ m/yr. Therefore the distance from the star to our sun is $(1.08 \times 10^{17}$ m$) \div (9.48 \times 10^{15}$ m/yr$) \approx 11.39$ light-years. **Options B and D are incorrect** because the calculation $(9.48 \times 10^{15}$ m/yr$) \div (1.08 \times 10^{17}$ m$)$ and 8.78×10^{-2} have units of $\frac{1}{year}$ instead of the correct unit years. **Option C is incorrect** because 0.114×10^{2} is not equivalent to 1.14×10^{2}.

2. The number of bacteria in a colony increases from 400 to 800 during 1 hour. What is the rate of growth?

A. 400% per hour

B. 200% per hour

C. 100% per hour

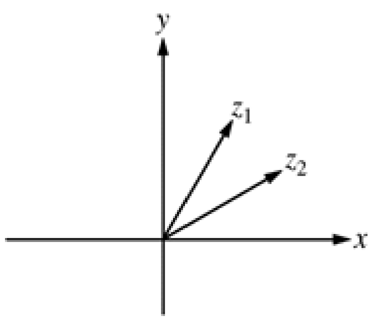
D. 50% per hour

Answer

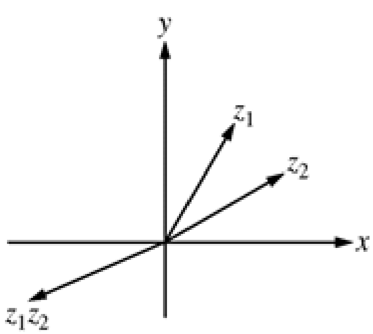
Option C is correct because the growth rate in 1 hour is $\left(\frac{800 - 400}{400}\right) \times 100 = \frac{400}{400} \times 100 = 100\%$. **Option A is incorrect** because the growth is 400 bacteria per hour, not 400%. **Options B and D are incorrect** because these growth rates are not supported by the data.

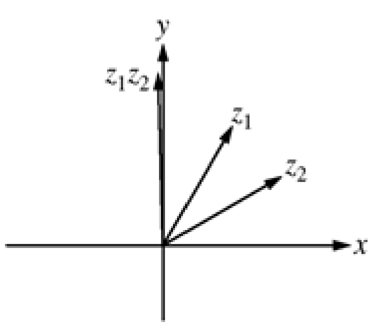
Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

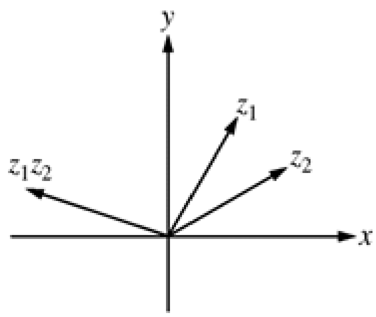
Use the figure below to answer the question that follows.

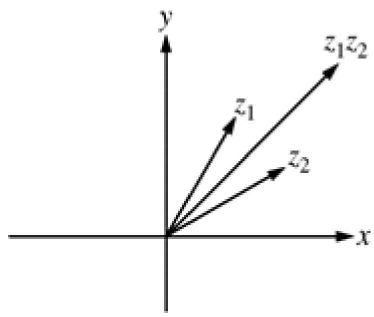


3. Two complex numbers, Z_1 and Z_2 are shown in the complex plane above. Which of the following graphs could depict the product Z_1Z_2 in the complex plane?

A. 

B. 

C. 

D. 

Answer

Option B is correct because the argument of the product of two complex numbers is the sum of their arguments. The sum of the arguments of the two given complex numbers is slightly more than $\frac{\pi}{2}$ radians. Only option B has the correct argument for the product. **Options A, C and D are incorrect** because the arguments of Z_1Z_2 shown are not the sum of the arguments of Z_1 and Z_2.

Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

4. Given that $n - 5$ and $n + 6$ are positive integers both divisible by prime number p, which of the following integers must also be divisible by p?

A. $n + 1$

B. $n - 30$

C. $n + 28$

D. $n - 11$

Answer

Option C is correct because p divides both $n - 5$ and $n + 6$, therefore p must also divide their difference, $(n + 6) - (n - 5) = 11$, and thus $p = 11$. Then p is also a factor of $n + 6 + 11k$ and $n - 5 + 11k$ for any integer k. Of the given options, only $n + 28 = n + 6 + 22 = n - 5 + 33$ has this form. **Options A, B and D are incorrect** because these integers cannot be written in the form $n + 6 + 11k$ or $n - 5 + 11k$, for some integer k.

5. On the first day of 2011 Asad deposited $500 into a savings account earning 3 percent annual interest, compounded annually at the end of the year. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2012, Asad deposited an additional $500 into the account, earning the same annual interest, compounded annually. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2013, Asad deposited an additional $500 into the account, earning the same annual interest, compounded annually. If no additional deposits or withdrawals were made, which of the following expressions represents the amount of money in Asad's account at the end of 2013?

A. $3[500 + 500(0.03)] + 500(0.06)$

B. $3[500 + 500(1.03) + 500(2.06)]$

C. $500(1.03) + 500(1.03)^2 + 500(1.03)^3$

D. $500(0.03) + 500(0.03)^2 + 500(0.03)^3$

Answer

Option C is correct because the first term represents the value of the third deposit, the second term represents the value of the second deposit after two years and the third term represents the value of the first deposit after three years. The sum of these terms gives the total amount of money in the account at the end of the third year. **Options A and B are incorrect** because the compounding of the interest is not correct. **Option D is incorrect** because the terms 500(0.03) give the value of the interest, not the principal and interest.

Domain II—Patterns and Algebra

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the equations below to answer the question that follows.

$$y = -3x - 2$$

$$y = \begin{cases} x + 2, & x \le 2 \\ 10 - 2x, & x > 2 \end{cases}$$

6. Which of the following are the points of intersection of the graphs in the *xy*-plane of the equations shown?

A. (–1, 1) only

B. (–1, –5) only

C. (–12, 34) only

D. (–1, 1) and (–12, 34)

Answer

Option A is correct because the graph of the line $y = -3x - 2$ intersects the graph of $y = x + 2$ at the point (–1, 1), and $-1 < 2$. The graph of the line $y = -3x - 2$ intersects the line $y = 10 - 2x$ at the point (–12, 34), but this point is not on the graph of the piecewise defined function because –12 is not greater than 2. **Option B is incorrect** because the graph of the line $y = -3x - 2$ does not intersect the graph of $y = x + 2$ at the point (–1, –5). **Option C**

is incorrect because the graph of the line $y = -3x - 2$ intersects the line $y = 10 - 2x$ at the point (–12, 34), but this point is not on the graph of the piecewise defined function because –12 is not greater than 2. **Option D is incorrect** because there is only one point of intersection between the two graphs.

Competency 008—The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

7. Which of the following is a vertical asymptote of the graph of $y = 3 + \log_2(x - 4)$?

A. $y = 3$

B. $y = -4$

C. $x = -4$

D. $x = 4$

Answer

Option D is correct because the graph of $y = \log_2 x$ has a vertical asymptote of the line $x = 0$, the y-axis. The function $y = 3 + \log_2(x - 4)$ is a transformation of the graph of $y = \log_2 x$, with a vertical shift of 3 and a horizontal shift of 4. Thus, the vertical asymptote shifts horizontally from $x = 0$ to $x = 4$. **Options A and B are incorrect** because the lines $y = 3$ and $y = -4$ are horizontal and cannot be vertical asymptotes. **Option C is incorrect** because the graph of $y = \log_2 x$ is shifted horizontally by the transformation to the right, not the left.

Competency 010—The teacher understands and solves problems using differential and integral calculus.

8. $\frac{d}{dx}\int_0^x \sin t \cos t \, dt =$

A. $\frac{\sin^2 x \cos^2 x}{4}$

B. $\cos 2x$

C. $\frac{\sin^2 x}{2}$

D. $\sin x \cos x$

Answer

Option D is correct because this is a direct application of the Fundamental Theorem of Calculus. **Options A, B and C are incorrect** because these are incorrect applications of the formulas for finding derivatives and calculating integrals.

Domain III—Geometry and Measurement

Competency 011—The teacher understands measurement as a process.

9. Which of the following definite integrals represents the length of the curve $y = x^2 + 1$ between the points (0, 1) and (2, 5)?

A. $\int_0^2 (1 + 4x^2)\,dx$

B. $\int_0^2 \sqrt{1 + 4x^2}\,dx$

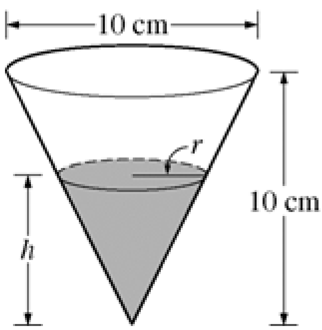
C. $\int_0^2 \sqrt{1 + x^2}\,dx$

D. $\pi \int_0^2 4x^2\,dx$

Answer

Option B is correct because the length of the curve $y = f(x)$ between $x = a$ and $x = b$ is found by the integral $\int_a^b \sqrt{1 + (f'(x))^2}\,dx$. Thus, because $y' = 2x$, the length of the curve is $\int_0^2 \sqrt{1 + 4x^2}\,dx$ over this interval. **Options A, C and D are incorrect** because they are not correct applications of the length of a curve formula.

Use the figure below to answer the question that follows.



10. An inverted circular cone contains water, as shown. If the height, h, of the water is 8 centimeters, what is the radius, r, of the surface of the water, in centimeters?

A. 8

B. 6

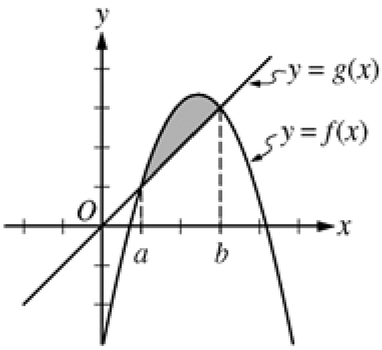
C. 4

D. 2

Answer

Option C is correct because the ratio of r to h is equal to the ratio of the radius of the cone to the height of the cone, by the similarity of triangles. Because the radius of the cone is $\frac{10}{2} = 5$, and $\frac{r}{8} = \frac{5}{10}$, then $r = 4$. **Options A, B and D are incorrect** because a radius of these lengths would not satisfy the required ratio.

Use the figure below to answer the question that follows.



11. Let *f* and *g* be continuous functions defined on the interval [*a*, *b*], as shown. Which of the following can be used to find the area of the shaded region?

A. $\int_a^b f(x)dx - \int_a^b g(x)dx$

B. $\int_a^b g(x)dx - \int_b^a f(x)dx$

C. $\int_a^b \pi(f(x) - g(x))^2\,dx$

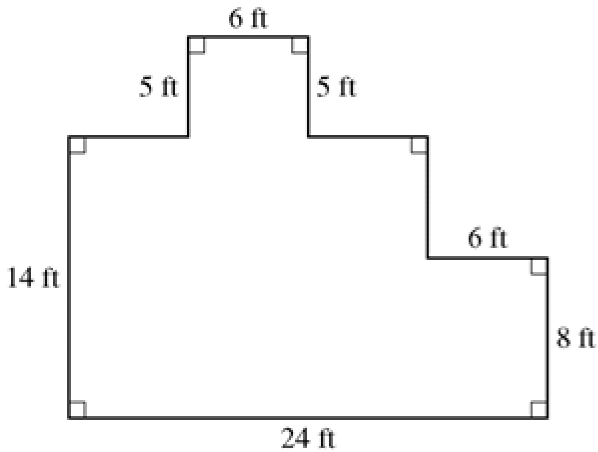
D. $2\pi\int_a^b x(f(x) - g(x))\,dx$

Answer

Option A is correct because the shaded area is found by subtracting the area under the graph of *g* from *a* to *b*, $\int_a^b g(x)dx$, from the area under the graph of *f* from *a* to *b*, $\int_a^b f(x)dx$. **Option B is incorrect** because the expression yields the negative of the area of the shaded region. **Option C is incorrect** because the expression represents the volume of the solid of revolution generated by revolving the shaded region about the *x*-axis. **Option D is incorrect** because the expression represents the volume of the solid of revolution generated by revolving the shaded region about the *y*-axis.

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

Use the figure below to answer the question that follows.



12. The floor plan of Nick's den is shown. Nick wishes to purchase square tiles that are 12 inches on each side to tile the entire floor. To allow for possible breakage, he plans to purchase more tiles to cover an additional 10 percent of the floor area. How many tiles will Nick purchase?

A. 330 tiles

B. 363 tiles

C. 366 tiles

D. 402 tiles

Answer

Option B is correct because the floor area consists of three rectangles, 8 × 24, 6 × 18, and 5 × 6, for a total area of 330 square feet. The tiles are each 1 square foot. Allowing for 10 percent breakage, Nick needs to purchase an additional 33 = 0.10(330) tiles for a total of 363 tiles. **Option A is incorrect** because 330 square feet is the total area, and does not include extra tiles required in case of breakage. **Option C is incorrect** because 366 is the incorrect area derived from 14 × 24 plus 5 × 6. **Option D is incorrect** because 402 is derived from the incorrect area of 366, plus 10 percent.

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

13. Let the vectors *i* and *j* be given by *i* = (1, 0) and *j* = (0, 1). A person walks 80 feet due east, and then turns counter-clockwise 40° and walks an additional 60 feet in that direction. Which of the following vectors describes the person's final position relative to his or her starting position?

A. (60cos40°)*i* + (80 + 60cos40°)*j*

B. (60cos40°)*i* + (80 + 60sin40°)*j*

C. (80 + 60sin40°)*i* + (60sin40°)*j*

D. (80 + 60cos40°)*i* + (60sin40°)*j*

Answer

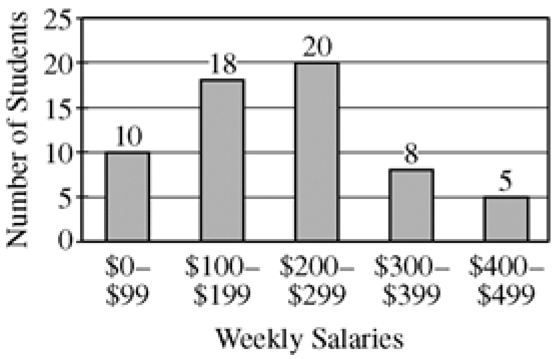
Option D is correct because the first vector expressed in component form is 80*i* and the second vector is (60cos40°)*i* + (60sin40°)*j*. The resultant is then (80 + 60cos40°)*i* + (60sin40°)*j*. **Options A and B are incorrect** because the component 80*j* would result from walking due north rather than walking due east. **Option C is incorrect** because the second direction vector is (60cos40°)*i* + (60sin40°)*j* not (60sin40°)*i* + (60sin40°)*j*.

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

Use the chart below to answer the question that follows.

SALARIES OF STUDENTS WORKING PART-TIME



14. The bar graph shown represents the distributions of weekly salaries, rounded to the nearest dollar, for a group of students with part-time jobs. Which of the following statements is true?

A. The range of the weekly salaries is $499.

B. The median weekly salary is between $240 and $260.

C. The average weekly salary is between $167 and $267.

D. The mode of the weekly salaries is between $200 and $299.

Answer

Option C is correct because the chart shown is a histogram, and each bar displays a range of salaries. The chart shows that there are salaries of 61 students recorded. To calculate the possible average salary, find the weighted mean of the lowest possible salary and the highest possible salary, as shown in the chart. The weighted mean of the lowest possible salary is

$$\frac{10(0) + 18(100) + 20(200) + 8(300) + 5(400)}{61} = \frac{10{,}200}{61} = \$167.21$$

The weighted mean of the highest possible salary is

$$\frac{10(99) + 18(199) + 20(299) + 8(399) + 5(499)}{61} = \frac{16{,}239}{61} = \$266.21$$

Thus, the average weekly salary is greater than \$167 and less than \$267. **Option A is incorrect** because the range of the salaries cannot be determined from the information given; the highest and lowest salaries are not indicated on the chart. **Option B is incorrect** because the median salary is determined by ordering the salaries from least to greatest and finding the salary in the middle. There are 61 salaries, so the 31^{st} salary is the median and will lie in the bar labeled \$200–\$299. **Option D is incorrect** because the mode is the salary that occurs most often in the list of salaries, but the individual values are not provided.

Competency 016—The teacher understands concepts and applications of probability.

15. Mr. Allen's math class consists of 15 boys and 15 girls. Each day he randomly selects 3 students to present homework problems on the blackboard. What is the probability that the three chosen students will be all boys? (Note: ${}_nC_r = \frac{n!}{(n-r)!r!}$ denotes combinations of n objects taken r at a time, and ${}_nP_r = \frac{n!}{(n-r)!}$ denotes permutations of n objects taken r at a time.)

A. $\frac{{}_{15}P_3}{{}_{30}C_3}$

B. $\frac{{}_{15}C_3}{{}_{15}P_3}$

C. $\frac{{}_{15}C_3}{{}_{30}C_3}$

D. $\frac{{}_{15}P_3}{{}_{15}C_3}$

Answer

Option C is correct because ${}_{30}C_3$ gives the number of ways to choose 3 students from the 30-person class, and ${}_{15}C_3$ gives the number of ways of choosing 3 boys from among the 15 boys. Thus, the ratio $\frac{{}_{15}C_3}{{}_{30}C_3}$ gives the probability that the teacher will choose 3 boys. **Options A, B and D are incorrect** because the numbers given do not correctly count the number of students to select.

Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.

16. A packing company packages approximately 10,000 packages of noodles each day. The packages have a mean weight of 16 ounces with a standard deviation of 0.2 ounces. An inspector selects multiple samples of 25 packages each and computes the mean weight of each sample. Which of the following is the best estimate of the standard deviation of the mean weights of the samples?

A. 0.02

B. 0.04

C. 0.2

D. 1.0

Answer

Option B is correct because the standard deviation of the sample means should be $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{25}} = 0.04$. **Options A, C and D are incorrect** because the numbers given are not derived for the relevant formula.

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

17. Mr. Matthews' precalculus class is studying even and odd functions. Mr. Matthews gave a function to a group of students in his class and asked them to investigate its symmetry without the use of a graphing calculator. Katie, Shayan and Diego are given the function $f(x) = \frac{x^3 + x}{|x| - 4}$, where $x \neq \pm 4$. Katie notes that $f(1) = -\frac{2}{3}$ and $f(-1) = \frac{2}{3}$. Shayan computes $f(3) = -30$ and $f(-3) = 30$. Diego discovers that $f(2) = -5$ and $f(-2) = 5$. After comparing notes, the three students conjecture that the function is odd because the graph exhibits symmetry with respect to the origin. Which of the following activities would enable the group to verify this conjecture?

A. The students could look for a counterexample by trying several values for x. If they are not able to find a counterexample, they can deduce that the function exhibits symmetry with respect to the origin and is therefore odd.

B. The students could evaluate the function at an additional eight pairs of values of x that are additive inverses and note that in each case, $f(-x) = -f(x)$. They could then deduce that the conjecture is true.

C. The students could simplify the algebraic expression for $f(-x)$ and establish that it is equal to $-f(x)$ for all values of x, $x \neq \pm 4$.

D. The students could simplify the algebraic expression for $f(-x)$ and establish that it is equal to $f(x)$ for all values of x, $x \neq \pm 4$.

Answer

Option C is correct because to establish deductively that the graph of a function is odd and has symmetry with respect to the origin, it suffices to establish that $f(-x) = -f(x)$ for all values of x in the domain of the function. **Options A and B are incorrect** because the students may select only points where the conjecture is true and miss points that are counterexamples. **Option D is incorrect** because it is the criteria for an even function, not an odd function.

Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

18. Let C be the set of all right-circular cylinders of radius 3. Let $v(x)$ be the volume of the cylinders in C as a function of height, $x \geq 0$. The graph of the function $y = v(x)$ in the xy-plane is

A. a horizontal line through the point $(0, 9\pi)$.

B. a line with slope 9π and y-intercept 0.

C. a parabola with vertex (0, 0) passing through the point (1, 9π).

D. a cubic polynomial passing through the points (0, 0), (1, 9π), and (3, 27π).

Answer

Option B is correct because the volume of a cylinder varies directly with the height when the radius is kept constant. The resulting graph will be a line with slope $(3^2)\pi = 9\pi$ in the first-quadrant of the *xy*-plane through the origin. **Option A is incorrect** because the line representing the volume of the cylinder has a non-zero slope of $(3^2)\pi = 9\pi$. **Options C and D are incorrect** because the volume of a cylinder varies directly with the height when the radius is kept constant and thus is not a polynomial of degree 2 or 3.

19. Bank A and Bank B each offer 5 percent interest rate on savings accounts, but Bank A compounds the interest annually and Bank B compounds the interest semiannually. If Samantha invests $10,000 in a savings account in Bank B, and makes no other transactions in her account, how much more interest will she earn in a year than she would have earned if she had invested the money in Bank A?

A. $ 5.00

B. $ 6.25

C. $ 7.25

D. $ 8.50

Answer

Option B is correct because if a principal of *P* dollars is invested at an annual interest rate *r*, compounded *n* times per year, and no further withdrawals from or deposits into the account are made, then the future value *A* after *t* years, is given by the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Because Bank A compounds interest annually, the future value of $10,000 invested in a savings account in Bank A after one year is $A = \$10{,}000.00\left(1 + \frac{.05}{1}\right)^{1} =$ $10,500.00. Bank B compounds interest semiannually, so that the future value of $10,000 invested at Bank B for one year is given by the formula $B = \$10{,}000.00\left(1 + \frac{.05}{2}\right)^{2} = \$10{,}506.25$. Thus, Samantha earns $10,506.25 − $10,500.00 = $6.25 more interest in one year by investing in a savings account in Bank B. **Options A, C and D are incorrect** because they are derived from incorrect applications of the formula.

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

20. Which of the following is a main purpose of formative assessments?

A. To provide feedback for teachers and students to improve the teaching and learning activities used for instruction

B. To summarize and record overall achievement at the end of an instructional unit

C. To evaluate the effectiveness of the curriculum

D. To identify students ready for promotion to the next level of a course

Answer

Option A is correct because formative assessments are used at the beginning of, or during, instruction with the aim of refining instructional practices. Thus, they provide feedback to the teachers and students, and are not intended to be reported. **Options B, C and D are incorrect** because they do not describe formative assessments.

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

21. The standard deviation of a set of data measurements is related to which of the following? Select <u>all</u> that apply.

A. Precision

B. Measurement repeatability

C. Systematic error

D. Reproducibility

Answer

Options A, B and D are correct because the standard deviation is a measure of precision, repeatability or reproducibility. **Option C is incorrect** because it affects the true or actual value of a quantity (i.e., the accuracy).

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

22. Scientific theories are based on which of the following? Select <u>all</u> that apply.

A. Observations

B. Experiments

C. Empirical generalizations

D. Personal beliefs and speculations

Answer

Options A, B and C are correct because theories are based on observations, experiments and empirical generalizations. **Option D is incorrect** because theories are not based on personal beliefs and speculations.

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

23. A ball of mass *m* is thrown horizontally with an initial speed u_0 from the top of a building that is height *h* above level ground. In the absence of air resistance, what is the speed of the ball when it strikes the ground?

A. u_0

B. $\sqrt{2gh}$

C. $u_0 + \sqrt{2gh}$

D. $\sqrt{u_0^2 + 2gh}$

Answer

Option D is correct because it correctly accounts for the vector nature of velocity. When the ball strikes the ground, it will have velocity components in both the horizontal and vertical directions. In the absence of air resistance, the external force in the horizontal direction is zero and the external force in the vertical direction is the downward force of gravity. Thus, the horizontal component of the ball's velocity, u_x, will remain constant throughout the motion at its initial value of u_0; that is, $u_x = u_0$. Also, the kinematic equations for constant acceleration, $y = \frac{1}{2}gt^2$ and $v = gt$, can be used to find that the vertical component of the velocity, u_y, when the ball strikes the ground is equal to $u_y = \sqrt{2gh}$. Thus, the speed of the ball when it strikes the ground is equal to the magnitude of the resultant of the two vectors, or $\sqrt{u_x^2 + u_y^2} = \sqrt{u_0^2 + 2gh}$. **Option A is incorrect** because it is equal to the horizontal component of the velocity only. **Option B is incorrect** because it is equal to the vertical component of the velocity only. **Option C is incorrect** because it is equal to the sum of the horizontal and vertical components of the velocity, which is not equal to the magnitude of the resultant of the two vectors.

24. Two satellites are each in a circular orbit around Earth at distances *R* and 2*R*, respectively, from Earth's center. If the satellite at distance *R* has an orbital period equal to *T*, the satellite at distance 2*R* must have an orbital period equal to

A. $2\sqrt{2}\,T$

B. $2T$

C. T

D. $\frac{1}{2}T$

Answer

Option A is correct because, according to Kepler's third law, the period $T_1 = T$ at radius $R_1 = R$ is related to the period T_2 at radius $R_2 = 2R$ by the equation $(\frac{T_2}{T_1})^2 = (\frac{R_2}{R_1})^3$, which gives $(\frac{T_2}{T})^2 = (\frac{2R}{R})^3 = 8$, or $T_2 = 2\sqrt{2}\,T$. **Options B, C and D are incorrect** because they do not agree with Kepler's third law.

25. A block of mass 2 kg and volume 4×10^{-4} m^3 is suspended from a thin wire and completely submerged in water. What is the tension in the wire? (Assume g = 10 m/s^2.)

A. 0

B. 4 N

C. 16 N

D. 20 N

Answer

Option C is correct because, according to Archimedes' principle, the buoyant force acting on the block is equal to the weight of the volume of water displaced by the block, or (1,000 kg/m^3)(4×10^{-4} m^3)(10 m/s^2) = 4 N. The tension in the wire is equal in magnitude to the apparent weight of the block which is equal to its actual weight minus the buoyant force, or 20 N − 4 N = 16 N. **Option A is incorrect** because it results from the assumption that the buoyant force is equal to the actual weight of the block. **Option B is incorrect** because it results from the assumption that the tension in the wire is equal to the buoyant force. **Option D is incorrect** because it results from the assumption that the tension in the wire is equal to the weight of the block.

Competency 026—The teacher understands the laws of motion.

26. A 40 kg box is pulled up a long, frictionless ramp by a 400 N force. If the ramp is inclined at an angle of 30° with respect to the horizontal, what is the magnitude of the acceleration of the block? (Assume g = 10 m/s^2.)

A. 0

B. 5 m/s^2

C. 10 m/s^2

D. 15 m/s^2

Answer

Option B is correct because, by Newton's second law, the acceleration of the block is equal to the net force acting on the block divided by the mass of the block. The net force F_{net} on the block is equal to the 400 N force acting upward along the ramp minus the component of the block weight acting downward along the ramp, or F_{net} = 400 N − (40 kg)(10 m/s^2)sin30° = 400 N − 200 N = 200 N. So F_{net} = 200 N.

Thus, by Newton's second law, the acceleration a of the block is equal to $a = \frac{F_{net}}{m} = \frac{200 \text{ N}}{40 \text{ kg}}$ = 5 m/s^2. **Option A is incorrect** because a net force is acting on the block along the ramp, which means the block is accelerating. **Option C is incorrect** because it does not account for the component of the block's weight along the ramp. **Option D is incorrect** because it results from using the wrong value for the net force acting on the block.

27. A 50 N force is being applied to a 10 kg block on a rough horizontal surface. If the coefficient of kinetic friction between the moving block and the surface is 0.25, what is the magnitude of the acceleration of the block? (Assume g = 10 m/s^2.)

A. 2.5 m/s^2

B. 5 m/s^2

C. 7.5 m/s^2

D. 10 m/s^2

Answer

Option A is correct because the net force acting on the moving block is equal to the applied force minus the force of kinetic friction, or
$F_{net} = 50\text{ N} - \mu_k Mg$
$F_{net} = 50\text{ N} - (0.25)(10\text{ kg})(10\text{ m/s}^2)$
$F_{net} = 50\text{ N} - 25\text{ N} = 25\text{ N}$. Thus, by Newton's second law, the acceleration of the block is equal to the net force divided by the mass of the block, or $\frac{F_{net}}{10\text{ kg}} = \frac{25\text{ N}}{10\text{ kg}} = 2.5\text{ m/s}^2$. **Option B is incorrect** because it does not account for the frictional force acting on the block. **Option C is incorrect** because it results from computing the net force as the sum, rather than the difference, of the applied force and the frictional force. **Option D is incorrect** because it is equal to the acceleration due to gravity, not to the acceleration of the block.

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

28. A simple pendulum of length L has a period T on Earth. If a simple pendulum is to have the same period T on Jupiter, where the gravitational force is 2.5 times as great as the gravitational force on Earth, what does the length of the pendulum have to be?

A. $2.5L$

B. $\sqrt{2.5}\,L$

C. L

D. $\frac{L}{2.5}$

Answer

Option A is correct because the period T of a simple pendulum is given by the equation $T = 2\pi\sqrt{\frac{L}{g}}$ and, if the period on Jupiter is to be the same as the period on Earth, then it follows that $\frac{L}{g} = \frac{L_{Jupiter}}{g_{Jupiter}} = \frac{L_{Jupiter}}{2.5g}$, or $L_{Jupiter}$ = $2.5L$. **Option B is incorrect** because it results from the assumption that the acceleration due to gravity on Jupiter is equal to $\sqrt{2.5}\,g$ instead of $2.5g$. **Option C is incorrect** because it results from the assumption that the length is unchanged, which would not give the same period on both planets. **Option D is incorrect** because it results from the assumption that the acceleration due to gravity on Jupiter is equal to $\frac{g}{2.5}$ instead of $2.5g$.

29. A planet moves in an elliptical orbit about a star. Which of the following is true about the orbital speed of the planet?

A. It is constant.

B. It varies and is greatest when the planet is closest to the star.

C. It varies and is greatest when the planet is farthest from the star.

D. It cannot be determined from known physical laws.

Answer

Option B is correct because, according to Kepler's second law, a planet sweeps out equal areas in equal times in its elliptical orbit about a star, which means that the speed of the planet must be greatest when it is closest to the star. **Options A and C are incorrect** because they are not in agreement with Kepler's second law. **Option D is incorrect** because the speed of the planet can be determined from known physical laws.

30. A coil of wire produces a magnetic field of magnitude *B*. If the number of turns per unit length in the coil is increased by a factor of 2, the magnitude of the magnetic field produced by the coil will be equal to which of the following?

A. $\frac{1}{2}B$

B. B

C. $2B$

D. $4B$

Answer

Option C is correct because the magnitude *B* of the magnetic field produced by a coil of wire with *n* turns per unit length is given by $B = \mu nI$, where μ is the magnetic permeability and *I* is the current in the coil. Thus, increasing *n* by a factor of 2 will increase the magnitude of the magnetic field by a factor of 2. **Options A, B and D are incorrect** because they result from assuming the wrong dependence of *B* on *n*.

Competency 028—The teacher understands applications of electricity and magnetism.

31. A 4 Ω resistor and a 12 Ω resistor are connected in parallel to a 12 V battery. The power dissipated by the circuit is equal to which of the following?

A. 48 W

B. 18 W

C. 9 W

D. 4 W

Answer

Option A is correct because two resistors, R_1 and R_2, connected in parallel are equivalent to a single resistor given by $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$, and, according to Ohm's law, the power P dissipated by the circuit is equal to $P = IV = \frac{V^2}{R_{eq}}$. Thus, $R_{eq} = \frac{(4\Omega)(12\Omega)}{4\Omega + 12\Omega} = \frac{48\Omega^2}{16\Omega} = 3\Omega$, which gives $P = \frac{(12\text{ V})^2}{3\Omega} = 48$ W. **Option B is incorrect** because it results from using the difference of the two resistors as the equivalent resistance to compute the power. **Option C is incorrect** because it results from using the sum of the two resistors as the equivalent resistance to compute the power. **Option D is incorrect** because it results from using V instead of V^2 to compute the power.

32. Which of the following uses mechanical energy to produce electrical energy?

A. A transformer

B. A motor

C. A generator

D. A battery

Answer

Option C is correct because a generator converts mechanical energy into electrical energy. **Option A is incorrect** because a transformer increases or decreases voltage through electromagnetic induction. **Option B is incorrect** because a motor converts electrical energy into mechanical energy. **Option D is incorrect** because a battery converts chemical energy into electrical energy.

Competency 029—The teacher understands the conservation of energy and momentum.

33. A railroad boxcar of mass M is moving at speed u along a straight horizontal track. It collides elastically with a second boxcar of the same mass M that is at rest. What are the magnitude of the total momentum and the total kinetic energy of the two boxcars immediately after the collision?

	Momentum	Kinetic Energy
A.	0	$\frac{1}{2}Mu^2$
B.	Mu	$\frac{1}{2}Mu^2$
C.	0	Mu^2
D.	Mu	Mu^2

Answer

Option B is correct because momentum and kinetic energy are conserved in an elastic collision. The magnitude of the momentum before the collision is equal to the momentum Mu of the moving boxcar, which means that the magnitude of the total momentum of the two boxcars immediately after the collision must also be equal to Mu. The kinetic energy before the collision is equal to the kinetic energy $\frac{1}{2}Mu^2$ of the moving boxcar, which means that the total kinetic energy of the two boxcars immediately after the collision must be equal to $\frac{1}{2}Mu^2$. **Option A is incorrect** because the magnitude of the momentum immediately after the collision is not zero (the boxcars are not at rest). **Option C is incorrect** because immediately after the collision the magnitude of the momentum is not zero (the boxcars are not at rest) and the total kinetic energy is not equal to two times the kinetic energy before the collision. **Option D is incorrect** because the total kinetic energy immediately after the collision is not equal to two times the kinetic energy before the collision.

Competency 030—The teacher understands the laws of thermodynamics.

34. The second law of thermodynamics is specifically concerned with which of the following?

A. The conservation of energy

B. The behavior of gases

C. The behavior of the entropy of a system

D. The molecular basis of temperature

Answer

Option C is correct because the second law of thermodynamics is a statement about the entropy of a system. **Option A is incorrect** because the first law of thermodynamics, not the second law of thermodynamics, is a statement of the law of conservation of energy. **Option B is incorrect** because the behavior of gases is a consequence of the kinetic theory of gases, not of the second law of thermodynamics. **Option D is incorrect** because the molecular basis of temperature is a consequence of the kinetic theory of gases, not of the second law of thermodynamics.

35. When 100 J of heat energy is added to a system, the system does 40 J of work. What is the increase in the internal energy of the system?

A. 40 J

B. 60 J

C. 100 J

D. 140 J

Answer

Option B is correct because, by the first law of thermodynamics, the internal energy increase is equal to the difference between the heat added to a system and the work done by the system, which in this case gives 100 J − 40 J = 60 J. **Option A is incorrect** because the increase in the internal energy of the system is not equal to the work done by the system. **Option C is incorrect** because the increase in the internal energy of the system is not equal to the heat added to the system. **Option D is incorrect** because the increase in the internal energy of the system is not equal to the sum of the heat added to the system and the work done by the system.

Competency 031—The teacher understands the characteristics and behavior of waves.

36. A light ray passes from air (n = 1) into glass (n = 1.5). If the incident angle is 30°, what is the angle of refraction?

A. 30°

B. 60°

C. $\sin^{-1}\frac{1}{3}$

D. $\sin^{-1}\frac{3}{4}$

Answer

Option C is correct because according to Snell's law, $n_{air}\sin 30° = n_{glass}\sin\theta_{glass}$, where θ_{glass} is the angle of refraction in glass. Thus, $\frac{1}{2} = 1.5\sin\theta_{glass}$, or $\theta_{glass} = \sin^{-1}\frac{1}{3}$. **Option A is incorrect** because the refracted ray in the glass must be bent toward the normal, which means that the angle of refraction must be less than 30°. **Option B is incorrect** because the angle of refraction must be less than 30°. **Option D is incorrect** because it contains a mathematical mistake.

37. Which of the following is characteristic of light but not of sound?

A. Dispersion

B. Constructive and destructive interference

C. Bending around an obstacle

D. Propagation in a vacuum

Answer

Option D is correct because light does not require a medium in which to propagate, whereas sound waves are disturbances in a medium. **Options A, B and C are incorrect** because all waves, including light and sound waves, exhibit dispersion, constructive and destructive interference and bending around an obstacle.

Competency 032—The teacher understands the fundamental concepts of quantum physics.

38. According to the Bohr model of the hydrogen atom, how does the energy of an electron depend on the principal quantum number n?

A. The energy of an electron is proportional to $\frac{1}{n^2}$.

B. The energy of an electron is proportional to $\frac{1}{n}$.

C. The energy of an electron is proportional to n.

D. The energy of an electron is proportional to n^2.

Answer

Option A is correct because, according to the Bohr model, the energy E_n of an electron in the nth energy level of the hydrogen atom is equal to $E_n = -\frac{13.6}{n^2}$ eV. **Options B, C and D are incorrect** because they each have the wrong dependence on n.

39. In the photoelectric effect, light is incident on the surface of a metal and causes photoelectrons to be emitted. For a given metal, which of the following is true about the energy of the emitted photoelectrons?

A. The energy of the photoelectrons increases when the frequency of the incident light increases.

B. The energy of the photoelectrons increases when the intensity of the incident light increases.

C. The energy of the photoelectrons increases when the wavelength of the incident light increases.

D. The energy of the photoelectrons is characteristic of the metal and independent of the incident light.

Answer

Option A is correct because increasing the frequency of the incident light increases the energy of the emitted photoelectrons. **Option B is incorrect** because increasing the intensity of the incident light increases the number of emitted photoelectrons but has no effect on the energy of the photoelectrons. **Option C is incorrect** because decreasing, not increasing, the wavelength of the incoming light increases the energy of the emitted photoelectrons. **Option D is incorrect** because the energy of the emitted photoelectrons is not independent of the incident light.

Domain IX—Science Learning, Instruction and Assessment

Competency 033—The teacher understands researched-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

40. Which of the following student responses is an example of correct conceptual understanding?

A. The mass of an object is equal to its weight.

B. The speed of light is greater than the speed of sound.

C. Heavy objects always sink in water.

D. The Moon and the Sun are the same size.

Answer

Option B is correct because the speed of light is much greater than the speed of sound. **Option A is incorrect** because mass is a measure of the amount of matter an object contains, while weight is the force of gravity on an object. **Option C is incorrect** because some heavy objects can float on water. **Option D is incorrect** because the Sun is much larger than the Moon.